

# MATH 0290: Homework 9

Spring 2014

Due at the beginning of class on Wednesday, Apr. 2

You must show all your work to receive full credit. You are encouraged to discuss the homework with other students, but you must write up your own solutions. If you have any questions about the homework, please contact me in person or at [alk92@pitt.edu](mailto:alk92@pitt.edu).

## Problem 1

For each of the following one-dimensional ODEs, do the following: 1) Find the fixed points, 2) Using the theorem from class about  $df/dx|_{x^*}$ , classify the stability of each of the fixed points if possible, and 3) for any fixed point you couldn't classify using  $df/dx|_{x^*}$ , draw the phase line and try to classify stability that way.

(a)  $x' = (x - 2)(x + 3)$

(b)  $x' = \frac{x(x-1)}{1+x^2}$

(c)  $x' = x^2(x - 2)$

(d)  $x' = \sin x - x$

## Problem 2

Assume  $\mathbf{x}' = A\mathbf{x}$ . For each of the following matrices  $A$ , do the following: 1) classify the type of fixed point that  $\mathbf{x} = 0$  is, just using the trace and determinant to find the eigenvalues; and 2) determine whether  $\mathbf{x} = 0$  is stable. You don't need to find eigenvectors or plot anything.

(a)  $A = \begin{pmatrix} -2 & -3 \\ 5 & 1 \end{pmatrix}$

(b)  $A = \begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix}$

(c)  $A = \begin{pmatrix} 3 & 6 \\ -1 & -2 \end{pmatrix}$

### Problem 3

Consider the nonlinear system:

$$\begin{aligned}x' &= y + x(3/4 - y^2) \\ y' &= 1 - x\end{aligned}$$

(a) Find the fixed points.

(b) Write the Jacobian matrix

$$J(x, y) = \begin{pmatrix} \partial f / \partial x & \partial f / \partial y \\ \partial g / \partial x & \partial g / \partial y \end{pmatrix}$$

(c) Using the eigenvalues of the Jacobian evaluated at each fixed point, determine whether each of the fixed points you find is stable, unstable, or undetermined given the eigenvalues.

### Problem 4

Consider the nonlinear system:

$$\begin{aligned}x' &= (x - 1)y \\ y' &= x + x^2 - 2y^2\end{aligned}$$

(a) Find the fixed points (hint: there are four).

(b) Write the Jacobian matrix

$$J(x, y) = \begin{pmatrix} \partial f / \partial x & \partial f / \partial y \\ \partial g / \partial x & \partial g / \partial y \end{pmatrix}$$

(c) Using the eigenvalues of the Jacobian evaluated at each fixed point, determine whether each of the fixed points you find is stable, unstable, or undetermined given the eigenvalues.