MATH 0290: Homework 8

Spring 2014

Due at the beginning of class on Wednesday, Mar. 26

You must show all your work to receive full credit. You are encouraged to discuss the homework with other students, but you must write up your own solutions. If you have any questions about the homework, please contact me in person or at `alk92@pitt.edu`.

Problem 1

Suppose:

$$A = \begin{pmatrix} -2 & 1 \\ 2 & -2 \end{pmatrix} \tag{1}$$

(a) Write the characteristic polynomial for A.

(b) Find the eigenvalues of A.

(c) Find the eigenvectors of A corresponding to the eigenvalues. Show your work.

(d) Sketch a phase plane picture of the system $\mathbf{x}' = A\mathbf{x}$ like we did in class. Include the eigenvectors, the direction of the solutions along the lines parallel to the eigenvectors, and a few representative solutions in between.

(e) Classify the system as a nodal sink, nodal source, center, spiral sink, spiral source, line of equilibria, or saddle.

Problem 2

Suppose:

$$A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} \tag{2}$$

(a) Write the characteristic polynomial for A.

(b) Find the eigenvalues of A.

(c) Find the eigenvectors of A corresponding to the eigenvalues.

(d) Sketch a phase plane picture of the system $\mathbf{x}' = A\mathbf{x}$ like we did in class. Include the eigenvectors, the direction of the solutions along the lines parallel to the eigenvectors, and a few representative solutions in between.

(e) Classify the system as a nodal sink, nodal source, center, spiral sink, spiral source, line of equilibria, or saddle.

Problem 3

Suppose:

$$A = \begin{pmatrix} 1 & 2 \\ -1 & -2 \end{pmatrix} \tag{3}$$

(a) Write the characteristic polynomial for A.

(b) Find the eigenvalues of A.

(c) Find the eigenvectors of A corresponding to the eigenvalues.

(d) Sketch a phase plane picture of the system $\mathbf{x}' = A\mathbf{x}$ like we did in class. Include the eigenvectors, the direction of the solutions along the lines parallel to the eigenvectors, and a few representative solutions in between.

(e) Classify the system as a nodal sink, nodal source, center, spiral sink, spiral source, line of equilibria, or saddle.

Problem 4

Let's return to the two-population model from a few lectures ago. We will let $r(t)$ be the number of rabbits and $f(t)$ the number of foxes. Note that I've switched c and d from the lecture to make things cleaner. Suppose:

$$\begin{aligned} r' &= ar - bf \\ f' &= cr + df \end{aligned}$$

(a) Write this in matrix-vector notation: $\mathbf{x}' = A\mathbf{x}$.

(b) Suppose $a = 1$, $c = 1$, $d = -1$. Assume that b is an unknown positive number. b can be interpreted as how much the foxes suppress the rabbit population growth.

(i) For what values of b will the population of foxes and rabbits oscillate?

(ii) Now suppose $b = 1/4$. Find the eigenvalues and eigenvectors. Using these, write the general solution to the problem in the form $C_1\mathbf{x_1}(t) + C_2\mathbf{x_2}(t)$. As $t \to \infty$, will there be more foxes or more rabbits (assuming $C_1 \neq 0, C_2 \neq 0$)?