MATH 0290: Homework 4

Spring 2014

Due at the beginning of class on Wednesday, Feb. 5

You must show all your work to receive full credit. You are encouraged to discuss the homework with other students, but you must write up your own solutions. If you have any questions about the homework, please contact me in person or at `alk92@pitt.edu`.

Problem 1

(A) (PBA 4.1) For each equation, decide whether it is linear or nonlinear. If the equation is linear, state whether it is homogeneous or inhomogeneous. You don't need to show any work to answer these questions.

(i) $t^2 y'' = 4y' - \sin t$

(ii) $ty'' + \sin t y' + (1 + y')y = 0$

(iii) $(1 - t^2)y'' = 3y$

(B) (PBA 4.1) Show by direct substitution that the given functions $y_1(t)$ and $y_2(t)$ are solutions of the given differential equation. Then, verify, again by direct substitution, that any linear combination $C_1 y_1(t) + C_2 y_2(t)$ is also a solution.

$$y'' + 4y' + 4y = 0; \;\; y_1(t) = e^{-2t}, \; y_2(t) = te^{-2t}$$

Problem 2

For each ODE, use the characteristic polynomial to determine λ_1, λ_2. All of these problems will have distinct characteristic roots. Based on these roots, answer whether $e^{\lambda_1 t}$ is a 1) decaying exponential, 2) growing exponential, 3) an oscillating function (with fixed magnitude of the oscillation), 4) a decaying oscillation, or 5) a growing oscillation. Do the same for $e^{\lambda_2 t}$.

(A) $y'' + 4y' + 10y = 0$

(B) $y'' = -10y$

(C) $-y'' + 2y' = 8y$

(D) $y'' + y' - 3y = 0$

Problem 3

(PBA 4.3) Find the solution to the given initial value problem.

(A) $y'' - 2y' - 3y = 0;\ y(0) = 2,\ y'(0) = -3$

(B) $y'' - 2y' + y = 0;\ y(0) = 3,\ y'(0) = 2$

(C) $y'' - 4y' + 13y = 0;\ y(0) = 4,\ y'(0) = 0$

Problem 4

Suppose we have an underdamped spring-mass system that is linear and homogeneous as discussed in class. We know such systems exhibit oscillations that decay in magnitude. You don't need to show any work to answer these questions.

(A) Does the presence of a damping term (c in the notation we used in class) *decrease* or *increase* the frequency of oscillations compared to the natural frequency ω_0?

(B) As the mass m increases, does the frequency of oscillations *increase* or *decrease*? Assume that the damping constant $c = \mu/2m$ is fixed and doesn't vary with m (this would be the case if μ was proportional to m).

(C) As the mass m increases, do the oscillations decay *more quickly* or *less quickly*?

Problem 5

We're going to plan a retirement. Suppose we have an account that will always pay 5% interest. We'll assume that we'll live for 20 years after retiring, and we want to be able to withdraw $10,000 a year for those 20 years. We'll also assume that we work for 40 years. How much do we need to save per year, assuming we save a constant amount each year? Let's solve this in two parts. (As always, you can assume that interest is continuously compounded and withdrawals/savings are done continuously).

(A) First let's find how much we need to retire with. So, given $r = 5\%$ and a withdrawal rate of $10,000 per year, what does $P(0)$ need to be so that $P(20)$ is not negative? This is our minimum retirement amount.

(B) Assuming that you save at a rate of s and start out with \$0, what should s be so that, after 40 years, you have the minimum retirement amount that you found in (A)?