

MATH 0290: Homework 10

Spring 2014

Due at the beginning of class on Wednesday, Apr. 9

You must show all your work to receive full credit. You are encouraged to discuss the homework with other students, but you must write up your own solutions. If you have any questions about the homework, please contact me in person or at alk92@pitt.edu.

Problem 1

Consider the following population dynamics system:

$$\begin{aligned}r' &= r(1 - r - s) \\s' &= \frac{3}{4}s(1 - \frac{4}{3}s - \frac{2}{3}r)\end{aligned}$$

For each of the following steps, perform the calculations and draw the results on a phase plane.

- (a) Find the fixed points.
- (b) Draw the nullclines and the direction of motion along them.
- (c) Determine the type of fixed point each one is by computing the Jacobian.
- (d) Using these results, draw the complete phase plane picture for this problem.

Problem 2

Suppose $f(x)$ is defined on $[-\pi, \pi]$ and is given by:

$$f(x) = x/\pi$$

Find the coefficients $a_0, a_1, a_2, \dots$ and $b_1, b_2, \dots$ for the Fourier series of $f(x)$.

Problem 3

Consider the following equation:

$$x'' - \mu(1 - x^2)x' + x = 0$$

This is known as the van der Pol equation. It represents an oscillator with nonlinear damping.

(a) Convert this second order ODE into a system of two first order ODEs. To do so, let $x' = y$. Then solve for $y' = x''$ in terms of x and y . You should end up with two equations of the form:

$$\begin{aligned}x' &= y \\ y' &= g(x, y)\end{aligned}$$

This is a useful way to turn any second order ODE into a planar system.

(b) Let $\mu = 2$. Find the fixed point of the system and determine what type it is (nodal/spiral source/sink, saddle, etc.).

(c) Let $\mu = 2$. Plot the phase plane using `pplane`. Make the axis limits go from -4 to 4 for both x and y . Include the nullclines (`Solutions > Show nullclines`). Also label the fixed point (`Solutions > Find an equilibrium point`). Include a couple trajectories. Turn in your plot. What type of solution, in addition to the fixed point, is visible in your plot?

Problem 4

Suppose $f(x)$ is defined on $[-\pi, \pi]$ and is given by:

$$f(x) = \begin{cases} -1, & x \in [-\pi, 0) \\ 1, & x \in [0, \pi] \end{cases}$$

Find the coefficients $a_0, a_1, a_2, \dots$ and $b_1, b_2, \dots$ for the Fourier series of $f(x)$.