MATH 0290: Homework 1

Spring 2014

Due at the beginning of class on Wednesday, Jan. 15

You must show all your work to receive full credit. You are encouraged to discuss the homework with other students, but you must write up your own solutions. If you have any questions about the homework, please contact me in person or at `alk92@pitt.edu`.

Problem 1

Find the general solution to the following first-order ordinary differential equations. If possible, give an explicit solution for the unknown function.

(A) $y' = e^{t+y}$

(B) $y' = \frac{\cos^2(y)}{t^2}$

(C) $r' = \frac{1}{e^r + r^2} + r't$

Problem 2

Draw the direction field for the following equation:

$$y' = t^2 y(y-1)$$

Make sure to label your axes. Using the geometrical intuition gained from your figure, answer the following questions and draw the corresponding solutions on your direction field:

(A) What is the limiting value of $y(t)$ as $t \to \infty$ if $y(0) = 1/2$?

(B) What is the limiting value of $y(t)$ as $t \to \infty$ if $y(-1) = -1$?

(C) What is the limiting value of $y(t)$ as $t \to \infty$ if $y(1) = 2$?

Problem 3

According to Newton's law of cooling, the change in temperature of an object is proportional to the difference between its temperature and that of the surrounding environment. That is, if $T(t)$ is the temperature of the object at time t and A is the temperature of the surrounding environment, then:

$$\frac{dT}{dt} = -k(T - A)$$

Due to a polar vortex, the temperature in your basement is constant at -20° C. At some point in time, you measure the temperature of some pipes in your basement to be 23° C. Two hours later, you measure it to be 15° C. How much longer will it take for the temperature of the pipes to reach freezing?

Problem 4

Solve the following initial value problem and find the interval of existence.

$$y' = \frac{1}{(1+x)\cos(y)}; \quad y(e-1) = 0$$

Problem 5

On the first day of class, we talked about population dynamics for rabbits and foxes. Let's pretend that there are only rabbits for now. We had started by writing that, if $R(t)$ is the number of rabbits, then $\frac{dR}{dt} = aR$. Let's suppose that the rabbits' rate of reproduction is also influenced by the season:

$$\frac{dR}{dt} = (0.2 + 0.1\cos t)R$$

where the cosine term reflects fluctuations in reproduction rate over time and $t \in (0, 2\pi)$ represents one season with $t = 0$ being Summer.

(A) Write the general solution to this problem.

(B) If $R(0) = 100$, how many *more* rabbits will be present at $t = 5\pi/2$ (which corresponds to next Fall) than there would be if there were no seasonal fluctuations (that is, if $\frac{dR}{dt} = 0.2R$)?