1 First order ODEs

Linear equations:

$$y' - a(t)y = f(t) \Rightarrow u(t) = e^{-\int a(t)dt}$$

Exact equations:

$$\frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 0$$

2 Variation of parameters

$$v_1 = -\int \frac{g(t)y_2}{y_1y_2' - y_1'y_2}dt, \qquad v_2 = \int \frac{g(t)y_1}{y_1y_2' - y_1'y_2}dt$$

3 Driven harmonic motion

Given $y'' + 2cy' + \omega_0^2 y = A\cos\omega t$, the transfer function is given by $H(\omega) = G(\omega)e^{i(\omega t + \phi(\omega))}$.

$$G(\omega) = \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4c^2\omega^2}}$$
$$-\tan\phi(\omega) = \frac{2c\omega}{\omega_0^2 - \omega^2}$$

4 Selected Laplace transforms

Function $u(t)$	Laplace transform $U(s) = \int_0^\infty e^{-st}u(t)dt$
$\alpha f(t) + \beta g(t)$	$\alpha\mathcal{L}[f] + \beta\mathcal{L}[g]$
$e^{ct}f(t)$	$F(s-c), F = \mathcal{L}[f]$
$tf(t)$	$-F'(s), F = \mathcal{L}[f]$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
e^{at}	$\frac{1}{s-a}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$y'(t)$	$s\mathcal{L}[y] - y(0)$
$y''(t)$	$s^2\mathcal{L}[y] - sy(0) - y'(0)$
$H(t-c)$	$\frac{e^{-cs}}{s}$
$H(t-c)f(t-c)$	$e^{-cs}\mathcal{L}[f]$
$f(t)$	$\frac{\mathcal{L}[f_T]}{1-e^{-Ts}}$ (if f has period T)
$\delta(t-c)$	e^{-cs}
$f * g$	$\mathcal{L}[f]\mathcal{L}[g]$

Here $f * g$ is defined as $\int_0^t f(u)g(t-u)du$.

Linear algebra and systems

Given $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\mathrm{Tr}A = a + d$, $\mathrm{Det}A = ad - bc$.

$$\lambda_{1,2} = \frac{1}{2}\left(\mathrm{Tr}(A) \pm \sqrt{\mathrm{Tr}^2(A) - 4\mathrm{Det}(A)}\right)$$

$$J(x,y) = \begin{pmatrix} \partial f/\partial x & \partial f/\partial y \\ \partial g/\partial x & \partial g/\partial y \end{pmatrix}$$

Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos\left(\frac{k\pi x}{L}\right) + b_k \sin\left(\frac{k\pi x}{L}\right)$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)dx; \quad a_k = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{k\pi x}{L}\right)dx; \quad b_k = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{k\pi x}{L}\right)dx$$