# MATH 0290: Topics for exam 2

Spring 2014

March 5, 2014

This is a short summary of the topics that you are required to know for exam 2. The material goes from Wednesday, 1/29 to Wednesday, 2/26.

## 1 Second order ODEs

1. Second order equations with constant coefficients: $y(t) = C_1 y_1(t) + C_2 y_2(t)$.

   Real distinct, complex conjugate, and repeated roots of characteristic polynomial

2. The method of undetermined coefficients for equations of the form $y'' + py' + qy = f(t)$, where $f(t)$ takes a form that is repeated under differentiation. General solution: $y(t) = y_p + y_h$. Here $y_p$ is the particular solution corresponding to the forcing, and $y_h$ is the general solution to the homogeneous ODE (written above).

   How to deal with the case where $f(t)$ is a solution of the homogeneous ODE: multiply your guess for $y_p$ by $t$ and solve for it.

3. The method of variation of parameters for equations of the form $y'' + py' + qy = f(t)$. In this case, $y_p = v_1 y_1 + v_2 y_2$. The formulas for $v_1$ and $v_2$ will be provided (see the handout). General solution takes a similar form as above.

4. The method of variation of parameters for equations of the form $y'' + p(t)y' + q(t)y = f(t)$, assuming that $y_1(t)$ and $y_2(t)$ are given.

5. Harmonic motion

   Damped, underdamped, and critically damped cases

   Driven harmonic motion and the transfer function (transfer function is provided on the handout)

## 2 Laplace transforms

1. How to take the Laplace transform and the inverse Laplace transform

   Using the properties of the Laplace transform to find $\mathcal{L}[f]$ or $\mathcal{L}^{-1}[F]$ (table is included in the handout).

You should be comfortable with partial fractions, completing the square, and in general the algebra required to put solutions in the correct form

2. Solving ODEs using the Laplace transform

   Direct approach: Find $Y(s)$ and invert it

   Splitting the solution into input-free (no forcing) and state-free ($y(0) = y'(0) = 0$) solutions

   Finding the impulse response function and writing the solution as a convolution: $e * f(t)$

3. Discontinuous forcing terms

   Writing discontinuous functions in terms of Heavisides

   Laplace transforms of Heavisides and their inverse (equation is provided on the handout)

   Periodic functions and their Laplace transforms (equation is provided on the handout)

   The $\delta$-function. How to take integrals involving it, and when those integrals are nonzero (depending on the bounds of the integral). Taking Laplace transforms.